

التفريغ 01

$$V_{n+1} = \frac{3^{n+1} \times 2U_n}{2U_n - 1}$$

$$V_{n+1} = \frac{3^n \times 3 \times 2U_n}{2U_n - 1}$$

$$V_{n+1} = \frac{6 \times 3^n U_n}{2U_n - 1}$$

$$V_{n+1} = 6 V_n$$

وهذا V_n متتالية هندسية أساسها 6

$$V_n = V_0 \cdot 6^n \Leftrightarrow V_n = -\frac{1}{3} \times 6^n$$

$$V_n = \frac{3^n U_n}{2U_n - 1} \Leftrightarrow V_n(2U_n - 1) = 3^n U_n - 0$$

$$\Leftrightarrow U_n(2V_n - 3^n) = V_n$$

$$\Leftrightarrow U_n = \frac{V_n}{2V_n - 3^n}$$

$$\Leftrightarrow U_n = \frac{-\frac{6^n}{3}}{-\frac{2 \times 6^n}{3} - \frac{3^{n+1}}{3}}$$

$$\Leftrightarrow U_n = \frac{6^n}{2 \times 6^n + 3^{n+1}}$$

$$U_n = \frac{2^1 \times 2^n}{3^1(2^{n+1} + 3)}$$

$$U_n = \frac{2^n}{2^{n+1} + 3}$$

$$\lim_{n \rightarrow +\infty} U_n = \lim_{n \rightarrow +\infty} \frac{2^n}{2^1(2 + \frac{3}{2^n})}$$

$$= \lim_{n \rightarrow +\infty} \frac{1}{2 + \frac{3}{2^n}}$$

$$= \frac{1}{2}$$

" $2 > 1$; لذا $\lim_{n \rightarrow +\infty} \frac{3}{2^n} = 0$ ؛ لذا $2^n = +\infty$ "

$$U_{n+1} = \frac{2U_n}{2U_n + 1} \quad (1)$$

$$U_{n+1} = \frac{2U_n + 1 - 1}{2U_n + 1}$$

$$U_{n+1} = 1 - \frac{1}{2U_n + 1}$$

- نبين أن $\forall n \in \mathbb{N} \quad 0 < U_n < \frac{1}{2}$

$$\text{من أجل } n=0 \quad 0 < U_0 = \frac{1}{5} < \frac{1}{2} \quad (v)$$

فنفترض أن $0 < U_n < \frac{1}{2}$

ونبين أن $0 < U_{n+1} < \frac{1}{2}$

$$0 < U_n < \frac{1}{2} \Rightarrow 1 < 2U_n + 1 < 2$$

$$\Rightarrow -1 < \frac{-1}{2U_n + 1} < -\frac{1}{2}$$

$$\Rightarrow 0 < U_{n+1} < \frac{1}{2}$$

وعليه $\forall n \in \mathbb{N} \quad 0 < U_{n+1} < \frac{1}{2}$ (2)

$$U_{n+1} - U_n = \frac{2U_n - 2U_n^2 - U_n}{2U_n + 1}$$

$$= \frac{U_n - 2U_n^2}{2U_n + 1}$$

$$= \frac{U_n(1 - 2U_n)}{2U_n + 1}$$

$$* \quad 0 < 1 - 2U_n < 1$$

$$0 < 2U_n + 1 < 1$$

$$U_{n+1} - U_n > 0$$

وعليه U_n متزايدة.

$$V_{n+1} = \frac{3^{n+1} U_{n+1}}{2U_{n+1} - 1} \quad (3)$$

$$V_{n+1} = \frac{3^{n+1} \frac{2U_n}{2U_n + 1}}{2 \times \frac{2U_n}{2U_n + 1} - 1}$$

$$(\widehat{BD}, \widehat{AC}) = \frac{\pi}{2} [2\pi]$$

$$\frac{|c-a|}{|d-b|} = 1$$

$$|c-a| = |d-b|$$

$$(\widehat{CA}, \widehat{AB}) = \frac{\pi}{2} [2\pi] \Rightarrow AC = BD$$

إذن ABCD مربع.

التمرين 03

$$z_1 = \frac{-3}{2} + \frac{\sqrt{3}}{2}i$$

$$|z_1| = \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{3}$$

$$\cos \theta = \frac{-3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}$$

$$\sin \theta = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$$

$$\theta = \frac{5\pi}{6}$$

$$z_1 = \left[\sqrt{3}; \frac{5\pi}{6} \right]$$

$$z_2 = 2 - 2i$$

$$|z_2| = \sqrt{4+4}$$

$$= 2\sqrt{2}$$

$$\cos \theta = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sin \theta = -\frac{2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\theta = -\frac{\pi}{4}$$

$$z_2 = \left[2\sqrt{2}; -\frac{\pi}{4} \right]$$

التمرين 02

$$\frac{b-a}{b-c} = \frac{5+i+1+i}{5+i-3-7i}$$

$$= \frac{6+2i}{2-6i}$$

$$= i$$

$$\left| \frac{b-a}{b-c} \right| = 1$$

$$|b-a| = |b-c| \Leftrightarrow AB = BC$$

$$(\widehat{CB}, \widehat{AB}) = \frac{\pi}{2} [2\pi]$$

إذن ABC مثلث متساوي الساقين

وقائم الزاوية في B.

$$ABCD \text{ متوازي أضلاع} \Leftrightarrow \vec{AB} = \vec{DC}$$

$$\Leftrightarrow \text{aff}(\vec{AB}) = \text{aff}(\vec{DC})$$

$$\Leftrightarrow b-a = c-d$$

$$\Leftrightarrow d = c-b+a$$

$$\Leftrightarrow d = 3+7i-5-i+1+i$$

$$d = 5i-3$$

$$\frac{c-a}{d-b} = \frac{3+7i+1+i}{5i-3-5-i}$$

$$= \frac{8i+4}{4i-8}$$

$$= \frac{2i+1}{i-2} = \frac{(1+2i)(-2-i)}{5}$$

$$= -\frac{5i}{5}$$

$$= -i$$

$\lim_{x \rightarrow +\infty} x+1 - \ln(x+3)$ حساب

$\lim_{n \rightarrow +\infty} n+1 - \ln(n+3) = \lim_{n \rightarrow +\infty} (n+3) \left(\frac{n+1}{n+3} - \frac{\ln(n+3)}{n+3} \right)$
 $= +\infty$

لأن $\lim_{n \rightarrow +\infty} \frac{n+1}{n+3} = 1$ و $\lim_{n \rightarrow +\infty} \frac{\ln(n+3)}{n+3} = 0$

$\lim_{x \rightarrow -3^+} f(x)$ حساب

$\lim_{n \rightarrow -3^+} n+1 - \ln(n+3) = +\infty$
 $\lim_{n \rightarrow -3^+} \ln(n+3) = -\infty$
 $\lim_{n \rightarrow -3^+} n+1 = -2$

$\lim_{n \rightarrow +\infty} f(n) = +\infty$

$\lim_{n \rightarrow +\infty} \frac{f(n)}{n} = \lim_{n \rightarrow +\infty} 1 + \frac{1}{n} - \frac{\ln(n+3)}{n}$
 $= \lim_{n \rightarrow +\infty} 1 + \frac{1}{n} - \frac{\ln(n(1+\frac{3}{n}))}{n}$
 $= \lim_{n \rightarrow +\infty} 1 + \frac{1}{n} - \frac{\ln(n)}{n} - \frac{\ln(1+\frac{3}{n})}{n}$
 $= 1$

لأن $\lim_{n \rightarrow +\infty} \frac{\ln(n)}{n} = 0$ و $\lim_{n \rightarrow +\infty} \frac{\ln(1+\frac{3}{n})}{n} = 0$

$\lim_{n \rightarrow +\infty} f(n) - n = \lim_{n \rightarrow +\infty} 1 - \ln(n+3) = -\infty$

"لأن $\lim_{n \rightarrow +\infty} \ln(n+3) = +\infty$ "

بأن f يقبل فرعاً شاملياً باتجاه $y=2$ المستقيم

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$Z_1 \times Z_2 = [2\sqrt{2} \times \sqrt{3}; \frac{5\pi}{6} - \frac{\pi}{4}]$
 $= [2\sqrt{6}; \frac{7\pi}{12}]$

تحديد الشكل الجبري Z_1, Z_2

$Z_1 \times Z_2 = (-\frac{3}{2} + \frac{\sqrt{3}}{2}i)(2 - 2i)$
 $= -3 + \sqrt{3} + i(6 + \sqrt{3})$

$\cos \frac{7\pi}{12} = \frac{\sqrt{3}-3}{2\sqrt{6}}$
 $= \frac{\sqrt{6}(\sqrt{3}-3)}{12}$

$= \frac{3\sqrt{2} - 3\sqrt{6}}{12}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

$\sin \frac{7\pi}{12} = \frac{6+\sqrt{3}}{2\sqrt{6}}$

$= \frac{\sqrt{6}(6+\sqrt{3})}{12}$

$= \frac{6\sqrt{6} + 3\sqrt{2}}{12}$

$= \frac{\sqrt{6} + \sqrt{2}}{4}$

التمرين 04

الجزء 1

تحديد D_f

$D_f = \{x \in \mathbb{R} / x+3 > 0\}$

$D_f = \{x \in \mathbb{R} / x > -3\}$

$D_f =]-3; +\infty[$

3- أ- حساب المشتقة:

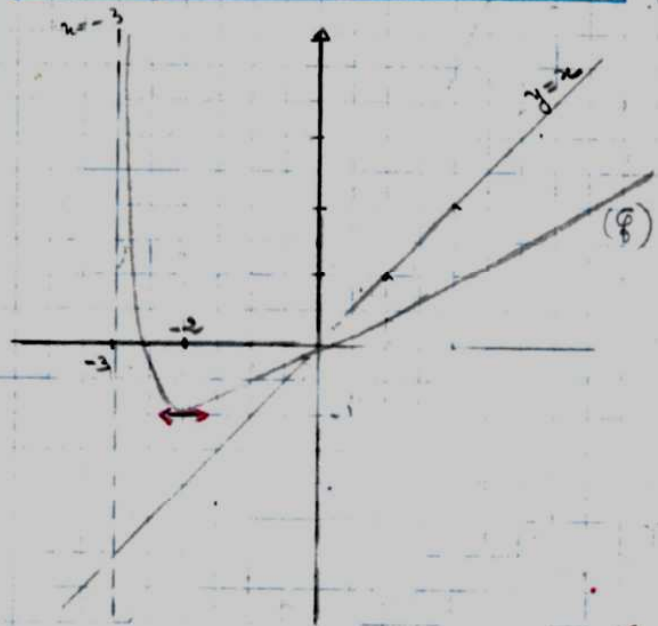
$$f'(x) = 1 - \frac{(x+3)'}{x+3} + 2'$$

$$f'(x) = -\frac{1}{x+3} + 1$$

$$f'(x) = \frac{x+3-1}{x+3}$$

$$f'(x) = \frac{x+2}{x+3}$$

x	-3	-2	$+\infty$
$f'(x)$		-	+
$f(x)$	$+\infty$	-1	$+\infty$



1) الجزء 2:

$$u_1 = f(u_0) \Leftrightarrow u_1 = 2 + 1 - \ln(5)$$

$$u_1 = 3 - \ln(5)$$

$$e < 5 \Leftrightarrow \ln e < \ln 5$$

$$\Leftrightarrow -1 > -\ln 5$$

$$\Leftrightarrow 2 > 3 - \ln 5$$

$$\Leftrightarrow u_0 > u_1$$

2- نبين ان $-2 \leq u_n \leq 2$ $\forall n \in \mathbb{N}$

$$(v) \quad -2 \leq u_0 = 2 \leq 2 \quad n=0 \text{ مزاحل}$$

$$-2 \leq u_n \leq 2 \text{ نفترض ان}$$

$$-2 \leq u_{n+1} \leq 2 \text{ ونريد ان}$$

لدينا $-2 \leq u_n \leq 2$ و f تزايدية

$$\text{على المجال } [-2, 2] \subset [-2, +\infty[$$

$$f(-2) \leq f(u_n) \leq f(2) \text{ إذن}$$

$$-2 \leq -1 \leq u_{n+1} \leq 2$$

$$-2 \leq u_{n+1} \leq 2$$

$$\forall n \in \mathbb{N} \quad -2 \leq u_n \leq 2 \text{ رافقه}$$

3- رتبة u_n :

$$u_0 > u_1 \text{ لدينا}$$

$$u_n > u_{n+1} \text{ نفترض ان}$$

$$u_{n+1} > u_{n+2} \text{ ولنن ان}$$

$$u_n > u_{n+2} \text{ لدينا}$$

و f تزايدية على المجال $[-2, 2]$

$$f(u_n) > f(u_{n+1})$$

$$u_{n+1} > u_{n+2}$$

$$\forall n \in \mathbb{N} \quad u_n > u_{n+1} \text{ إذن}$$

وعليه فإن u_n تناقصية

لدينا u_n تناقصية وعشوائية ب -2

إذن متقاربة

$$u_0 \in [-2, 2] \text{ و } f \text{ متصلة}$$

$$f([-2, 2]) = [-2, 3 - \ln(5)] \subset [-2, 2]$$

إذن نهاية u_n هي حل للمعادلة $f(x) = x$

$$f(x) = x \Leftrightarrow 1 - \ln(x+3) = 0$$

$$\ln(x+3) = \ln e$$

$$x = e - 3$$

$$\lim_{n \rightarrow +\infty} u_n = e - 3$$