

سلسلة 2	الحساب التكاملي	السنة 2 بكالوريا علوم تجريبية
تمرين 1: نعتبر التكاملين : $I = \int_0^{\frac{f}{2}} \frac{\cos(x)}{1+2\sin(x)} dx$ و $J = \int_0^{\frac{f}{2}} \frac{\sin(2x)}{1+2\sin(x)} dx$		
	$I = \int_0^{\frac{f}{2}} \frac{\cos(x)}{1+2\sin(x)} dx = \int_0^{\frac{f}{2}} \frac{1}{2} \frac{2\cos(x)}{1+2\sin(x)} dx = \frac{1}{2} \int_0^{\frac{f}{2}} \frac{(1+2\sin(x))'}{1+2\sin(x)} dx = \frac{1}{2} [\ln(1+2\sin(x))]_0^{\frac{f}{2}} = \frac{\ln(3)}{2}$	1
	$I + J = \int_0^{\frac{f}{2}} \frac{\cos(x)}{1+2\sin(x)} dx + \int_0^{\frac{f}{2}} \frac{\sin(2x)}{1+2\sin(x)} dx = \int_0^{\frac{f}{2}} \frac{\cos(x) + \sin(2x)}{1+2\sin(x)} dx$ $I + J = \int_0^{\frac{f}{2}} \frac{\cos(x) + 2\sin(x)\cos(x)}{1+2\sin(x)} dx = \int_0^{\frac{f}{2}} \frac{\cos(x)(1+2\sin(x))}{1+2\sin(x)} dx = \int_0^{\frac{f}{2}} \cos(x) dx = [\sin(x)]_0^{\frac{f}{2}} = 1$	2
	$J = (I + J) - I = 1 - \frac{\ln(3)}{2}$	3
تمرين 2: $J = \int_0^{\frac{f}{2}} \sin^2(x)\cos(x) dx$ و $I = \int_0^{\frac{f}{2}} \cos^3(x) dx$		
	$I + J = \int_0^{\frac{f}{2}} \cos^3(x) dx + \int_0^{\frac{f}{2}} \sin^2(x)\cos(x) dx = \int_0^{\frac{f}{2}} \cos^3(x) + \sin^2(x)\cos(x) dx$ $I + J = \int_0^{\frac{f}{2}} \cos(x)(\cos^2(x) + \sin^2(x)) dx = \int_0^{\frac{f}{2}} \cos(x) dx = [\sin(x)]_0^{\frac{f}{2}} = 1$	1
	$J = \int_0^{\frac{f}{2}} \sin^2(x)\cos(x) dx = \int_0^{\frac{f}{2}} \sin^2(x)(\sin(x))' dx = \left[\frac{1}{3} \sin^3(x) \right]_0^{\frac{f}{2}} = \frac{1}{3}$	2
	$I = (I + J) - J = 1 - \frac{1}{3} = \frac{2}{3}$	3
تمرين 3:		
	$\begin{cases} u'(x) = 1 \\ v'(x) = \cos(x) \end{cases} \text{ منه : } \begin{cases} u(x) = x \\ v(x) = \sin(x) \end{cases} \text{ نضع :}$ $\int_0^{\frac{f}{2}} x \cos(x) dx = \int_0^{\frac{f}{2}} u(x) v'(x) dx = [u(x)v(x)]_0^{\frac{f}{2}} - \int_0^{\frac{f}{2}} u'(x)v(x) dx = [x \sin(x)]_0^{\frac{f}{2}} - \int_0^{\frac{f}{2}} \sin(x) dx$ $\int_0^{\frac{f}{2}} x \cos(x) dx = [x \sin(x)]_0^{\frac{f}{2}} - [\cos(x)]_0^{\frac{f}{2}} = \left(\frac{f}{2} - 0 \right) - (0 - 1) = \frac{f}{2} + 1$	
	$\begin{cases} u'(x) = 2 \\ v'(x) = e^x \end{cases} \text{ منه : } \begin{cases} u(x) = 3 + 2x \\ v(x) = e^x \end{cases} \text{ نضع :}$ $\int_0^1 (3 + 2x)e^x dx = \int_0^1 u(x)v'(x) dx = [u(x)v(x)]_0^1 - \int_0^1 u'(x)v(x) dx = [(3 + 2x)e^x]_0^1 - \int_0^1 2e^x dx$ $= [(3 + 2x)e^x]_0^1 - [2e^x]_0^1 = (5e - 3) - (2e - 2) = 3e - 1$	
	$\begin{cases} u'(x) = 1 \\ v'(x) = \frac{1}{x} \end{cases} \text{ منه : } \begin{cases} u(x) = x \\ v(x) = \ln x \end{cases} \text{ نضع :}$ $\int_1^2 \ln(x) dx = \int_1^2 u'(x)v(x) dx = [u(x)v(x)]_1^2 - \int_1^2 u(x)v'(x) dx = [x \ln(x)]_1^2 - \int_1^2 x \times \frac{1}{x} dx$ $\int_1^2 \ln(x) dx = [x \ln(x)]_1^2 - \int_1^2 1 dx = [x \ln(x)]_1^2 - [x]_1^2 = (2 \ln(2) - 0) - (2 - 1) = 2 \ln(2) - 1$	

$$\begin{cases} u'(x) = \frac{1}{x^2} \\ v'(x) = \frac{1}{x} \end{cases} : \text{منه} \quad \begin{cases} u(x) = \frac{-1}{x} \\ v(x) = \ln x \end{cases} : \text{نضع}$$

$$\int_1^e \frac{\ln(x)}{x^2} dx = \int_1^e u'(x)v(x) dx = [u(x)v(x)]_1^e - \int_1^e u(x)v'(x) dx = \left[\frac{-\ln(x)}{x} \right]_1^e - \int_1^e \frac{-1}{x} \times \frac{1}{x} dx$$

$$\int_1^e \frac{\ln(x)}{x^2} dx = \left[\frac{-\ln(x)}{x} \right]_1^e + \int_1^e \frac{1}{x^2} dx = \left[\frac{-\ln(x)}{x} \right]_1^e + \left[\frac{-1}{x} \right]_1^e = \left(\frac{-1}{e} + 0 \right) + \left(\frac{-1}{e} + 1 \right) = \frac{-2}{e} + 1$$

$$J = \int_0^{\frac{f}{2}} \sin(x) e^x dx \quad , \quad I = \int_0^{\frac{f}{2}} \cos(x) e^x dx \quad : \text{تمرين 4}$$

$$J = \int_0^{\frac{f}{2}} \sin(x) e^x dx = \int_0^{\frac{f}{2}} \sin(x) (e^x)' dx = [\sin(x)e^x]_0^{\frac{f}{2}} - \int_0^{\frac{f}{2}} (\sin(x))' e^x dx$$

$$J = e^{\frac{f}{2}} - \int_0^{\frac{f}{2}} \cos(x) e^x dx = e^{\frac{f}{2}} - I$$

1

$$I = \int_0^{\frac{f}{2}} \cos(x) e^x dx = \int_0^{\frac{f}{2}} \cos(x) (e^x)' dx = [\cos(x)e^x]_0^{\frac{f}{2}} - \int_0^{\frac{f}{2}} (\cos(x))' e^x dx$$

$$I = -1 - \int_0^{\frac{f}{2}} -\sin(x) e^x dx = -1 + J$$

2

$$\begin{cases} J = e^{\frac{f}{2}} - I \\ I = -1 + J \end{cases} \Rightarrow \begin{cases} J = e^{\frac{f}{2}} - I \\ I = -1 + e^{\frac{f}{2}} - I \end{cases} \Rightarrow \begin{cases} J = e^{\frac{f}{2}} - \frac{-1 + e^{\frac{f}{2}}}{2} \\ I = \frac{-1 + e^{\frac{f}{2}}}{2} \end{cases} \Rightarrow \begin{cases} J = \frac{1 + e^{\frac{f}{2}}}{2} \\ I = \frac{-1 + e^{\frac{f}{2}}}{2} \end{cases}$$

3

تمرين 5 :

$$I = \int_0^1 x^2 e^x dx = \int_0^1 u(x)v'(x) dx$$

$$I = [u(x)v(x)]_0^1 - \int_0^1 u'(x)v(x) dx$$

$$I = [x^2 e^x]_0^1 - \int_0^1 2x e^x dx = e - \int_0^1 2x e^x dx$$

$$: \text{منه} \quad , \quad \begin{cases} u'(x) = 2x \\ v'(x) = e^x \end{cases} : \text{منه} \quad \begin{cases} u(x) = x^2 \\ v(x) = e^x \end{cases} : \text{نضع}$$

$$: \text{منه} \quad , \quad \begin{cases} u'(x) = 2 \\ v'(x) = e^x \end{cases} : \text{منه} \quad \begin{cases} u(x) = 2x \\ v(x) = e^x \end{cases} : \text{نضع مرة أخرى}$$

1

$$I = e - 2 \quad : \text{بالتالي} \quad , \quad \int_0^1 2x e^x dx = \int_0^1 u(x)v'(x) dx = [u(x)v(x)]_0^1 - \int_0^1 u'(x)v(x) dx$$

$$\int_0^1 2x e^x dx = [2x e^x]_0^1 - \int_0^1 2 e^x dx = 2e - [2e^x]_0^1 = 2e - (2e - 2) = 2$$

$$F'(x) = x^2 e^x \Leftrightarrow (2x + a)e^x + (x^2 + ax + b)e^x = x^2 e^x$$

$$F'(x) = x^2 e^x \Leftrightarrow e^x(2x + a + x^2 + ax + b) = x^2 e^x$$

$$F'(x) = x^2 e^x \Leftrightarrow 2x + a + x^2 + ax + b = x^2$$

$$F'(x) = x^2 e^x \Leftrightarrow (2 + a)x + a + b = 0$$

$$F'(x) = x^2 e^x \Leftrightarrow \begin{cases} 2 + a = 0 \\ a + b = 0 \end{cases} \Leftrightarrow \begin{cases} a = -2 \\ b = 2 \end{cases}$$

أ

2

$$I = \int_0^1 x^2 e^x dx = [(x^2 - 2x + 2)e^x]_0^1 = e - 2$$

ب

تمرين 6 : $n \in \mathbb{N}^*$ $I_n = \int_1^e t^n \ln(t) dt$

نضع : $\begin{cases} u(t) = \frac{t^{n+1}}{n+1} \\ v(t) = \ln(t) \end{cases}$ منه : $\begin{cases} u'(t) = t^n \\ v'(t) = \frac{1}{t} \end{cases}$ منه :

$$I_n = \int_1^e t^n \ln(t) dt = \int_1^e u'(t)v(t) dt = [u(t)v(t)]_1^e - \int_1^e u(t)v'(t) dt = \left[\frac{t^{n+1}}{n+1} \ln(t) \right]_1^e - \int_1^e \frac{t^{n+1}}{n+1} \times \frac{1}{t} dt \quad 1$$

$$I_n = \frac{e^{n+1}}{n+1} - \frac{1}{n+1} \int_1^e t^n dt = \frac{e^{n+1}}{n+1} - \frac{1}{n+1} \left[\frac{t^{n+1}}{n+1} \right]_1^e = \frac{e^{n+1}}{n+1} - \frac{1}{n+1} \left(\frac{e^{n+1}}{n+1} - \frac{1}{n+1} \right)$$

$$I_n = \frac{e^{n+1}}{n+1} \left(1 - \frac{1}{n+1} \right) + \frac{1}{(n+1)^2} = \frac{e^{n+1}}{n+1} \times \frac{n}{n+1} + \frac{1}{(n+1)^2}$$

نعلم أن : $\lim_{n \rightarrow +\infty} \frac{e^{n+1}}{n+1} = +\infty$ و $\lim_{n \rightarrow +\infty} \frac{n}{n+1} = \lim_{n \rightarrow +\infty} \frac{n}{n} = 1$ و $\lim_{n \rightarrow +\infty} \frac{1}{(n+1)^2} = 0$ ، إذن : $\lim_{n \rightarrow +\infty} I_n = +\infty$ 2

رياضيات النجاح أذ سمير لخريسي